Overview:
When solving problems it is helpful to follow the guideline below (R.E.S.T.):

1. **Read** the problem
   - Basic question: *What do I need to find out?*
   - The aim is to understand what the problem is about. Diagrams and/or tables are helpful most times.

2. **Explore** the problem
   - Basic questions: *What information am I given (implicitly or explicitly)? What am I not given?*
   - It is useful to assign a symbol to the quantity you want to find. It is best to try to keep to using only one symbol as much as possible. Diagrams and/or tables also help here.

3. **Strategize** (Develop a strategy) & **Solve** using the strategy
   - Basic question: *How can I use what I have to get what I want?*
   - This usually involves the use of formulas or equations. What formula or series of formulas (or equations) relate what you have to what you want?

4. **Test** your answer (when possible) to see if it makes sense.
   - Basic question: *Does my answer make sense?*
   - If the answer doesn’t make sense, then chances are it is incorrect. Also, **be careful that you answered the question asked.** The problem is not solved until you answer the question asked.

Tables:
Tables many times simplify the information you are given. Usually (but not always) they follow the format below:

- **1st Column:** What type of quantity (e.g. Type of mixture; savings bonds, etc.)
- **Middle Columns:** The amount (or number) of the quantities you’re given. That is, the information you have.
- **Last Column:** Usually a quantity that combines those in the middle columns. For example, the items in the middle columns may multiply to give the quantity in the last column.

Note how the tables are used in each of the exercises that follow (pages 3 to 6).
Special Formulas:
1. Interest

\[ I = PRT; \]
where \( I \) = Simple interest;
\( P \) = Principal (original amt.)
\( R \) = Rate (usually per year/annum);
\( T \) = Time (usually in years).

N.B. If the time period for the rate is per year, then the time must be in years. Likewise for months, days, and any time period.

2. Mixture Problems

\[ A = CV \]
Where
\( A \) = Amount of pure substance in a mixture/solution,
\( C \) = Concentration of mixture/solution
\( V \) = Volume of mixture/solution

3. Uniform Motion

\[ s = vt \]
where \( s \) = distance/displacement
\( v \) = velocity
\( t \) = time
Following are four examples of solving problems. All are taken from the textbook. The source line tells the location of the problem in the College Algebra textbook.

1. Simple Interest

**Problem:** Betsy, a recent retiree, requires $6000 per year in extra income. She has $50,000 to invest and can invest in B-rated bonds paying 15% per year or in a certificate of deposit (CD) paying 7% per year. How much money should be invested in each to realize exactly $6000 in interest per year?

*Source: Algebra & Trigonometry 6th Ed. by Michael Sullivan, Exercises 1.2, #27, p. 103.*

Total Principal (starting amount) = $50,000
Total Interest = $6,000
Interest rate on B-rated bonds = 15% per year = (15)/(100) = 0.15 per year
Interest rate on CD = 7% per year = 7/(100) = 0.07 per year
Time = 1 year
We wish to find the amount invested in B-rated bonds and the amount invested in CD to give a total interest of $6,000

We make the amount invested in B-rated bonds = X
Since the rest is invested in CD, then amount invested in CD = 50,000 – X

If we find X, then we can get our answer.
Note that I = PRT

Table:

<table>
<thead>
<tr>
<th>Type of Bond</th>
<th>Principal ($)</th>
<th>Rate (per year)</th>
<th>Time (years)</th>
<th>Interest ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-rate</td>
<td>X</td>
<td>15% = 0.15</td>
<td>1</td>
<td>0.15X</td>
</tr>
<tr>
<td>CD</td>
<td>50,000 – X</td>
<td>7% = 0.07</td>
<td>1</td>
<td>0.07(50,000 – X)</td>
</tr>
<tr>
<td>Total</td>
<td>50,000</td>
<td>Not applicable</td>
<td>1</td>
<td>6,000</td>
</tr>
</tbody>
</table>

**Calculation:**
Total Interest = Interest on B-rate + Interest on CD
6000 = 0.15X + 0.07(50,000 – X)
1. = 0.15X + 3500 – 0.07X
2. = 0.08X
X = 31,250

So she invested $31,250 in the B-rate bonds and ($50,000 - $31,250) = $18,750 in CD.

**Recheck:** Use the interest formula to calculate the interest on the amount put in the B-rate bonds and the interest on the amount put in the CD and add the answers. You should get the total interest, which is $6,000.
2. Mixture Problems

Problem: In a chemistry laboratory the concentration of one solution is 10\% hydrochloric acid (HCl) and that of a second solution is 60\% HCl. How many milliliters (mL) of each should be mixed to obtain 50 mL of a 30\% HCl solution?

Source: Algebra & Trigonometry 5\textsuperscript{th} Ed. by Michael Sullivan, Section 2.2, Example 6.

Mixture A = 10\% HCl
Mixture B = 60\% HCl
Final Mixture = 30\% HCl
Volume of final mixture = 50 mL

We wish to find the volume of mixture A and the volume of mixture B that combine to give 50 mL of the Final Mixture.

Let the volume of Mixture A = X
Then, since Mixture A and mixture B combine to give 50 mL,
  volume of Mixture B = 50 – X

Note that the amount of pure HCl in a mixture = CV (concentration \cdot volume)

<table>
<thead>
<tr>
<th>Mixture</th>
<th>Volume [V] (in mL)</th>
<th>Concentration [C] of HCl</th>
<th>Amount of pure HCl in mixture (mL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixture A</td>
<td>X</td>
<td>10% = 0.10</td>
<td>0.10X</td>
</tr>
<tr>
<td>Mixture B</td>
<td>50 – X</td>
<td>60% = 0.60</td>
<td>0.60(50 – X)</td>
</tr>
<tr>
<td>Final Mixture</td>
<td>50</td>
<td>30% = 0.30</td>
<td>0.30(50) = 15</td>
</tr>
</tbody>
</table>

Calculation:
Amount of acid in final mixture = amount in mixture A + amount in mixture B

\[ 15 = 0.10X + 0.60(50 – X) \]
\[ 15 = 0.10X + 30 – 0.60X \]
\[ -15 = -0.50X \]
\[ X = 30 \]

So the amount of mixture A used = 30 mL.
The amount of mixture B used = 50 – 30 = 20 mL.

Recheck: Use the formula to calculate how much acid is in 30 mL of mixture A and how much is in 20 mL of mixture B. That amount should be 30\% of the final mixture. (Do the recheck on your own.)
3. **Uniform Motion**

**Problem:** A motorboat heads upstream on a river that has a current of 3 miles per hour. The trip upstream takes 5 hours, and the return trip takes 2.5 hours. What is the speed of the motorboat? (Assume that the motorboat maintains a constant speed relative to the water.)

*Source: Algebra & Trigonometry 6th Ed. by Michael Sullivan, Exercises 1.2, #36, p. 103.*

Speed of river current = 3 mph
Time to go upstream = 5 hrs.
Time to go downstream = 2.5 hrs.
Note that the distance upstream and the distance downstream are the SAME.

We wish to find the speed of the motorboat relative to the water (that is, if the current wasn’t helping or hindering it, how fast would the boat go?)

Let the speed of the motorboat relative to the water = \( X \)
Note that distance = speed \( \times \) time

Water current goes downstream

Upstream \( \Rightarrow \) \( X – 3 \) (current hinders motorboat by 3 mph)

Downstream \( \Rightarrow \) \( X + 3 \) (current helps motorboat by 3 mph)

<table>
<thead>
<tr>
<th>Direction</th>
<th>Time (hrs.)</th>
<th>Speed (mph)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream</td>
<td>5</td>
<td>( X – 3 )</td>
<td>( 5(X – 3) )</td>
</tr>
<tr>
<td>Downstream</td>
<td>2.5</td>
<td>( X + 3 )</td>
<td>( 2.5(X + 3) )</td>
</tr>
</tbody>
</table>

**Calculation:**
Distance upstream = Distance downstream

\[
5(X – 3) = 2.5(X + 3)
\]

\[
5x – 15 = 2.5x + 7.5
\]

\[
5x – 2.5x = 15 + 7.5
\]

\[
2.5x = 22.5
\]

\[
x = 9
\]

So the speed of the boat relative to the water is 9 mph.

**Recheck:** Find the speed upstream and downstream by replacing \( X \) in the table with 9. Calculate the distances upstream and downstream (by multiplying upstream speed by the time it takes and likewise with the downstream speed). Both distances should be the same.
4. Constant Rate Jobs

**Problem:** Patrice, by himself, can paint four rooms in 10 hours. If he hires April to help, they can do the same job together in 6 hours. If he lets April work alone, how long will it take her to paint four rooms?


Time it takes Patrice to paint four rooms = 10 hrs
Time it takes Patrice and April together to paint four rooms = 6 hrs

We wish to find how long it takes April to paint the four rooms by herself.

Let the time April takes to paint the rooms by herself = t
Note that if it takes t hrs to do a task, then (1/t) of the task is done in 1 hr. (e.g. If it takes me 2 hours to paint a room, then ½ of the room will be painted in 1 hour.)

<table>
<thead>
<tr>
<th></th>
<th>Time to paint 4 rooms (hrs.)</th>
<th># of rooms painted in 1 hr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patrice</td>
<td>10</td>
<td>4/10 = 0.4</td>
</tr>
<tr>
<td>April</td>
<td>t</td>
<td>4/t</td>
</tr>
<tr>
<td>Patrice and April</td>
<td>6</td>
<td>4/6</td>
</tr>
</tbody>
</table>

**Calculation:**
Work done by Patrice and April together in 1 hr. = work done by Patrice alone in 1 hr
+ work done by April alone in 1 hr.

\[
\frac{4}{6} = \frac{4}{10} + \frac{4}{t} \\
\frac{4}{6} - \frac{4}{10} = \frac{4}{t} \\
\frac{20}{30} - \frac{12}{30} = \frac{4}{t} \\
\frac{8}{30} = \frac{4}{t} \\
t = 4 \cdot \frac{30}{8} \\
t = 15
\]

So April takes 15 hours to paint the four rooms on her own.

**Recheck (A bit tricky):**
It takes April 15 hrs to paint 4 rooms. Patrice paints 6 rooms in the same 15 hours (he paints 4 in 10 hours – that’s 2 rooms every 5 hours.) In all they both paint 4 + 6 = 10 rooms in 15 hours. To find out how long they both take to paint 4 rooms is simple from here:
10 rooms = 15 hours
1 room = 1.5 hours (divide by 10)
4 rooms = 1.5 \cdot 4 = 6 hours (multiply by 4)