GRAPHS OF FUNDAMENTAL FUNCTIONS
The following are fundamental functions whose stated properties and graphs you must know.

1. The Constant Function
\[ y = f(x) = c \]

**Properties:**
(I) Domain: \( x \in (-\infty, \infty) \)
(II) Range: \( y \in \{c\} \) or \( y = c \).
(III) y-intercept: \((0, c)\)
\[ x \text{-intercept: None except for } y = f(x) = 0 \] (In this case the x-axis is the graph)
(IV) Constant over \( x \in (-\infty, \infty) \), that is, always constant
(V) Symmetry: Even (y-axis symmetry)
(VI) End Behavior:
\[ \text{As } x \to -\infty, y = c \]
\[ \text{As } x \to \infty, y = c \]
(VII) No asymptote.

2. The Identity Function
\[ y = f(x) = x \]

**Properties:**
(I) Domain: \( x \in (-\infty, \infty) \)
(II) Range: \( y \in (-\infty, \infty) \).
(III) y-intercept: \((0,0)\); \( x \text{-intercept: } (0,0) \)
(IV) Increasing over \( x \in (-\infty, \infty) \), that is, always increasing
(V) Symmetry: Odd (origin symmetry)
(VI) End Behavior:
\[ \text{As } x \to -\infty, y \to -\infty \]
\[ \text{As } x \to \infty, y \to \infty \]
(VII) No asymptote.
3. The Absolute Value Function

\[ y = f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \]

**Properties:**

(I) Domain: \( x \in (-\infty, \infty) \)

(II) Range: \( y \in [0, \infty) \).

(III) y-intercept: (0,0); x-intercept: (0,0)

(IV) Decreasing over \( x \in (-\infty, 0) \). Increasing over \( x \in (0, \infty) \).

(V) Symmetry: Even (y-axis symmetry)

(VI) End Behavior: As \( x \to -\infty \), \( y \to \infty \)

As \( x \to \infty \), \( y \to \infty \)

(VII) No asymptote.

4. The Square Function

\[ y = f(x) = x^2 \]

**Properties:**

(I) Domain: \( x \in (-\infty, \infty) \)

(II) Range: \( y \in [0, \infty) \).

(III) y-intercept: (0,0); x-intercept: (0,0)

(IV) Decreasing over \( x \in (-\infty, 0) \). Increasing over \( x \in (0, \infty) \).

(V) Symmetry: Even (y-axis symmetry)

(VI) End Behavior: As \( x \to -\infty \), \( y \to \infty \)

As \( x \to \infty \), \( y \to \infty \)

(VII) No asymptote.

5. The Cube Function

\[ y = f(x) = x^3 \]

**Properties:**

(I) Domain: \( x \in (-\infty, \infty) \)

(II) Range: \( y \in (-\infty, \infty) \).

(III) y-intercept: (0,0); x-intercept: (0,0)

(IV) Increasing over \( x \in (-\infty, \infty) \); that is, always increasing

(V) Symmetry: Odd (origin symmetry)

(VI) End Behavior: As \( x \to -\infty \), \( y \to -\infty \)

As \( x \to \infty \), \( y \to \infty \)

(VII) No asymptote.
6. The Square-Root Function

\[ y = f(x) = x^{1/2} = \sqrt{x} \]

**Properties:**

(I) **Domain:** \( x \in [0, \infty) \)

(II) **Range:** \( y \in [0, \infty) \)

(III) **y-intercept:** (0,0); **x-intercept:** (0,0)

(IV) **Increasing** over \( x \in (0, \infty) \).

(V) **Symmetry:** None

(VI) **End Behavior:**

- As \( x \to 0^+ \), \( y \to 0 \)
- As \( x \to \infty \), \( y \to \infty \)

(VII) No asymptote.

7. The Reciprocal Function

\[ y = f(x) = \frac{1}{x} \]

**Properties:**

(I) **Domain:** \( x \in (-\infty, 0) \cup (0, \infty) \). That is, all real numbers except \( x = 0 \).

(II) **Range:** \( y \in (-\infty, 0) \cup (0, \infty) \). That is, all real numbers except \( y = 0 \).

(III) **y-intercept:** None; **x-intercept:** None

(IV) **Decreasing** over \( x \in (-\infty, 0) \) and over \( x \in (0, \infty) \).

(V) **Symmetry:** Odd (origin symmetry)

(VI) **End Behavior:**

- As \( x \to -\infty \), \( y \to 0 \)
- As \( x \to 0^- \) (approaches 0 from the left), \( y \to -\infty \)
- As \( x \to 0^+ \) (approaches 0 from the right), \( y \to \infty \)
- As \( x \to \infty \), \( y \to 0 \)

(VII) **Vertical asymptote:** \( x = 0 \) (y-axis); **Horizontal asymptote:** \( y = 0 \) (x-axis)
8. The Exponential Function

\[ y = f(x) = e^x \]

**Properties:**

(I) Domain: \( x \in (-\infty, \infty) \)

(II) Range: \( y \in (0, \infty) \).

(III) y-intercept: (0,1); x-intercept: None

(IV) Increasing over \( x \in (-\infty, \infty) \); that is, always increasing

(V) Symmetry: None

(VI) End Behavior:
- As \( x \to -\infty \), \( y \to 0 \)
- As \( x \to \infty \), \( y \to \infty \)

(VII) Horizontal asymptote: \( y = 0 \) (the x-axis). No vertical asymptote.

9. The Natural Logarithm Function

\[ y = f(x) = \ln(x) \]

**Properties:**

(I) Domain: \( x \in (0, \infty) \)

(II) Range: \( y \in (-\infty, \infty) \).

(III) y-intercept: None; x-intercept: (1,0)

(IV) Increasing over \( x \in (0, \infty) \); that is, always increasing

(V) Symmetry: None

(VI) End Behavior:
- As \( x \to 0^- \), \( y \to -\infty \)
- As \( x \to \infty \), \( y \to \infty \)

(VII) Vertical asymptote: \( x = 0 \) (the y-axis). No horizontal asymptote.

Note:

\( y = e^x \) and \( y = \ln(x) \) are inverse functions.

If two functions are inverses of each other then the domain of one is the range of the other and vice versa. For example, if \((2, -3)\) is a point on a function, then \((-3, 2)\) is a point on its inverse.

To get the graph of the inverse of a function from the graph of the function, simply reflect the graph about the line \( y = x \).

So if you start out with \( y = e^x \), you can get the graph of \( y = \ln(x) \), simply reflect the graph of \( y = e^x \) about the line \( y = x \).