MAC 1105: Quiz 3

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the discriminant and determine the number and type of roots of each equation.

1) \(16x^2 + 8x + 1 = 0\)
   \(b^2 - 4ac = 8^2 - 4(16)(1) = 0\)
   A) \(D = 0\), real equal roots
   B) \(D = -32\), complex roots
   C) \(D = 32\), real unequal roots

Find the power of \(i\).

2) \(i^{14} = \) \(\boxed{-1}\)
   A) \(-i\)
   B) \(-1\)
   C) 1
   D) \(i\)

Solve the equation by the square root property.

3) \((5x + 2)^2 - 10 = 0\) \(\Rightarrow (5x + 2)^2 = 10\)
   A) \(\left\{ -\frac{12}{5}, \frac{8}{5} \right\}\)
   B) \(\left\{ \frac{-2 - \sqrt{10}}{5}, \frac{-2 + \sqrt{10}}{5} \right\}\)
   C) \(\left\{ \frac{\sqrt{10} - 2}{5}, \frac{\sqrt{10} + 2}{5} \right\}\)
   D) \(\left\{ \frac{2 - \sqrt{10}}{5}, \frac{2 + \sqrt{10}}{5} \right\}\)

Determine the constant term that must be added to the expression to make it a perfect square.

4) \(x^2 + \frac{1}{6}x\)
   \(\left(\frac{1}{6} \cdot \frac{1}{2}\right)^2 = \left(\frac{1}{12}\right)^2 = \frac{1}{144}\)
   A) 144
   B) \(\frac{1}{36}\)
   C) \(\frac{1}{12}\)
   D) \(\frac{1}{144}\)
MAC 1105: Quiz 4

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

1. Solve the equation by making an appropriate substitution.
   \[ x^4 - 4x^2 - 45 = 0 \]
   \[ p = x^2 \]
   \[ p^2 - 4p - 45 = 0 \]
   \[ (p - 9)(p + 5) = 0 \]
   \[ p = 9 \quad \text{or} \quad p = -5 \]
   \[ x^2 = 9 \quad \Rightarrow \quad x = \pm 3 \]
   \[ x^2 = -5 \quad \Rightarrow \quad x = \pm i\sqrt{5} \]
   \[ \text{A) } [-9, 5] \]
   \[ \text{C) } [3, i\sqrt{5}] \]
   \[ \text{B) } [-\sqrt{5}, \sqrt{5}, -3i, 3i] \]
   \[ \text{D) } [-3, 3, -i\sqrt{5}, i\sqrt{5}] \]

2. Solve the equation by multiplying both sides by the LCD.
   \[ 1 - \frac{8}{x} - \frac{48}{x^2} = 0 \]
   \[ x^2 \quad \Rightarrow \quad x^2 - 8x - 48 = 0 \quad \Rightarrow \quad (x - 12)(x + 4) = 0 \]
   \[ x = 12, x = -4 \]
   \[ \text{A) } [-12, 4] \]
   \[ \text{B) } [12, -4] \]
   \[ \text{C) } [12, 4] \]
   \[ \text{D) } [-12, -4] \]

3. Solve the equation.
   \[ \sqrt{8x + 33} = x \]
   \[ 8x + 33 = x^2 \]
   \[ 0 = x^2 - 8x - 33 \]
   \[ 0 = (x - 11)(x + 3) \]
   \[ x = 11 \quad \text{or} \quad x = -3 \]
   \[ \text{A) } [-3, 11] \]
   \[ \text{B) } [-\frac{33}{7}] \]
   \[ \text{C) } [11] \]
   \[ \text{D) } \emptyset \]
Ex. 1.5 (p. 144). Solve each equation. Check your solutions.

**NOTE:** When solving radical equations:
1. Isolate the radical on one side of the equals.
2. Square both sides. Be sure to FOIL.
   - NEVER distribute exponents over addition nor subtraction.
3. Solve the remaining equation using normal methods.
4. If there are more than one radicals, first isolate one, then square both sides (FOIL), then isolate the remaining radical, and then square both sides again. Then solve as usual.
5. Remember: ALWAYS check your answers for extraneous solutions.

\[ 26. \quad x - \sqrt{6x+7} = 0 \]

\[ x = \sqrt{6x+7} \]

\[ x^2 = (\sqrt{6x+7})^2 \]

\[ x^2 = 6x + 7 \]

\[ \sqrt{3x+4} = -1 \]

\[ (\sqrt{3x+4})^2 = (-1)^2 \]

\[ 3x + 4 = 1 \]

\[ 3x = -3 \]

\[ x = -1 \] (X)

Check:

\[ \sqrt{3(-1)+4} = -1 \]

\[ 1 = -1 \] (X)

No soln: \( \emptyset \)

\[ x - 7 = 0 \]

\[ x = 7 \] (V)

or

\[ x + 1 = 0 \]

\[ x = -1 \] (X)
34. \( x + \sqrt{x+1} = 5 \)
\[
-x - x
\]
\[
\sqrt{x+1} = 5 - x
\]
\[
(\sqrt{x+1})^2 = (5-x)^2
\]
\[
x+1 = (5-x)(5-x)
\]
\[
x+1 = 25 - 10x + x^2
\]
\[
-x - 1 - x
\]
\[
0 = x^2 - 11x + 24
\]
\[
0 = (x-3)(x-8)
\]
\[
x-3 = 0 \text{ or } x-8 = 0
\]
\[
x = 3 \quad \text{or} \quad x = 8
\]
Soln: \{3, 8\}

42. \( \sqrt{3x+5} + \sqrt{6x+3} = 3 \)
\[
(\sqrt{3x+5})^2 = (3 - \sqrt{6x+3})^2
\]
\[
3x+5 = (3 - \sqrt{6x+3})(3 - \sqrt{6x+3})
\]
\[
3x+5 = 9 - 3\sqrt{6x+3} - 3\sqrt{6x+3} + (\sqrt{6x+3})^2
\]
\[
3x+5 = 9 - 6\sqrt{6x+3} + 6x + 3
\]
\[
-6x - 9 -9
\]
\[
-3x - 7 = -6\sqrt{6x+3}
\]
Multiply by \(-1\)
\[
(\sqrt{6x+3})^2 = 6\sqrt{6x+3}
\]
\[
(3x+7)(3x+7) = 6^2(\sqrt{6x+3})^2
\]
\[
9x^2 + 42x + 49 = 36(6x+3)
\]
\[
9x^2 + 42x + 49 = 216x + 108
\]
\[
x = \frac{174 \pm \sqrt{(-174)^2 - 4(9)(-59)}}{2(9)}
\]
\[
x = \frac{174 \pm \sqrt{32400}}{18}
\]
\[
x = \frac{174 \pm 180}{18}
\]
Soln: \{-\frac{1}{3}, \frac{1}{3}\}
50.) \(81y^4 + 1 = 18y^2\)

\[81y^4 - 18y^2 + 1 = 0\]

Let \(u = y^2\), so \(u^2 = y^4\)

So \(81u^2 - 18u + 1 = 0\)

\[u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[u = \frac{18 \pm \sqrt{(-18)^2 - 4(81)(1)}}{2(81)}\]

\[u = \frac{18 \pm \sqrt{(-18)^2 - 4(81)(1)}}{2(81)} = \frac{18 \pm \sqrt{0}}{162} = \frac{1}{9}\]

Resubstitute: \(y^2 = \frac{1}{9}\)

\[\sqrt{y^2} = \pm \sqrt{\frac{1}{9}} \Rightarrow y = \pm \frac{1}{3}\]

\(\text{Soln}: \{ \pm \frac{1}{3} \}\)

44.) \(x - 3\sqrt{x} + 2 = 0\)

Let \(m = \sqrt{x}\) so \(m^2 = x\)

\[m^2 - 3m + 2 = 0\]

\((m-1)(m-2) = 0\)

\(m-1 = 0 \text{ or } m-2 = 0\)

\(m = 1 \text{ or } m = 2\)

\(\sqrt{x} = 1 \text{ or } \sqrt{x} = 2\)

\((\sqrt{x})^2 = 1^2 \text{ or } (\sqrt{x})^2 = 2^2\)

\(x = 1 \checkmark \text{ or } x = 4 \checkmark\)

\(\text{Soln}: \{ 1, 4 \}\)
58.) \((x^2+2)^2 - 5(x^2+2) - 6 = 0\)

Let \(m = x^2+2\). So \(m^2 = (x^2+2)^2\)

\[m^2 - 5m - 6 = 0\]
\[(m-6)(m+1) = 0\]
\(m-6 = 0\) or \(m+1 = 0\)
\(m = 6\) or \(m = -1\)

\(x^2 + 2 = 6\) or \(x^2 + 2 = -1\)
\(-2\) \(-2\)
\[x^2 = 4\] or \[x^2 = -3\]
\(x = \pm 2\) or \(x = \pm \sqrt{-3}\)

Solutions: \(\{\pm 2, \pm \sqrt{3}\}\)

52.) \((x^2-3) - 4\sqrt{x^2-3} - 12 = 0\)

Let \(p = \sqrt{x^2-3}\). So \(p^2 = x^2-3\)

\[p^2 - 4p - 12 = 0\]
\[(p - 6)(p + 2) = 0\]
\(p - 6 = 0\) or \(p + 2 = 0\)
\(p = 6\) or \(p = -2\)

\(\sqrt{x^2-3} = 6\) or \(\sqrt{x^2-3} = -2\)

\(x^2 - 3 = 36\) or \(x^2 - 3 = 4\)
\(x^2 = 39\) or \(x^2 = 7\)
\(x = \pm \sqrt{39}\) or \(x = \pm \sqrt{7}\)

Solutions: \(\{\pm \sqrt{39}\}\)
64.) \((x^2+2x)^2 - 8(x^2+2x) + 15 = 0\)

Let \(n = (x^2+2x)\). So \(n^2 = (x^2+2x)^2\)

\[n^2 - 8n + 15 = 0\]
\[(n-3)(n-5) = 0\]

\(n-3 = 0\) or \(n-5 = 0\)

\(n = 3\) or \(n = 5\)

\(x^2 + 2x = 3\) or \(x^2 + 2x = 5\)

\(x^2 + 2x - 3 = 0\) or \(x^2 + 2x - 5 = 0\)

\((x+3)(x-1) = 0\)

\(x+3 = 0\) or \(x-1 = 0\)

\(x = -3\) or \(x = 1\)

\(x = \frac{-2 \pm \sqrt{4 - 4(1)(-5)}}{2}\)

\(x = \frac{-2 \pm \sqrt{24}}{2}\)

\(x = \frac{-2 \pm 2\sqrt{6}}{2}\)

\(x = -1 \pm \sqrt{6}\)

**CHECK**
Sec. 1.7: Polynomial and Rational Inequalities

Solving Polynomial Inequalities:
1. Write in standard form (that is, get everything on one side and zero (0) on the other side of the inequality:
   \[ x^2 - 8x + 7 > 0 \]

2. Solve the corresponding equation.
   \[ x^2 - 8x + 7 = 0 \]
   \[(x-1)(x-7) = 0 \]
   \[ x = 1 \quad \text{and} \quad x = 7 \]

3. Use the solutions from Step 2 as "markers" on the Real Number Line.

4. Choose numbers before, between and after the "markers" and evaluate the polynomial expression (in standard form) at those numbers. Indicate the signs in the corresponding regions on the number line.

5. Choose the regions that satisfy the inequality in standard form. Write answers in interval notation.

Solution: \((-\infty, 1) \cup (7, \infty)\)
Ex. 1.7 (p. 165) Solve the inequality. Give answers in interval notation.

6.) \((x - 4)(x + 1) \leq 0\)

\(\text{Solve: } (x - 4)(x + 1) = 0\)
\(x = 4 \text{ or } x = -1\)

\(\text{Sln: } [-1, 4]\)

18.) \(4x^2 - 2x - 2 < 0\)

\(\text{Solve: } 4x^2 - 2x - 2 = 0\)
\(2(2x^2 - x - 1) = 0\)
\(2 \left( \frac{2x^2 - 2x + x - 1}{x - 1} \right) = 0\)
\(2 (2x(x - 1) + 1(x - 1)) = 0\)
\(2 (2x + 1) (x - 1) = 0\)
\(x = \frac{1}{2} \text{ or } x = 1\)

\(\text{Sln: } (-\frac{1}{2}, 1)\)

22.) \((x + 3)^2 > 9\)

\(\text{Solve: } (x + 3)^2 - 9 > 0\)
\((x + 3)^2 = 9\)
\((x + 3) = \pm 3\)
\(x = -3 \pm 3\)
\(x = -3 + 3 = 0 \text{ or } x = -3 - 3 = -6\)

\(\text{Sln: } (-\infty, -6) \cup (0, \infty)\)
Ex. 1.7 (p. 165) Solve the inequality. Give answers in interval notation.

26) \(3x(x+2)(x-2) > 0\)

\[\text{solve: } 3x(x+2)(x-2) = 0\]
\[x = 0, \quad x = -2, \quad x = 2\]

\[\begin{array}{cccccc}
& & & - & & + \\
-2 & & & 0 & & 2
\end{array}\]

\[\text{Soln: } (-\infty, 0) \cup (2, \infty)\]

Note: If an equation does not give real number solutions, then the equation will either always be positive or always negative for any real number we use.

32) \(2x^2 \geq -5\)

\[\begin{array}{ccc}
2x^2 + 5 & \geq & 0 \\
\text{Not real numbers}
\end{array}\]

\[\text{solve: } 2x^2 + 5 = 0\]
\[2x^2 = -5\]
\[x^2 = -\frac{5}{2}\]
\[x = \pm \sqrt{-\frac{5}{2}} = \pm \frac{i\sqrt{5}}{2}\]

\[x = 0: \quad 2(0^2) + 5 = 5 > 0\]

So \(2x^2 + 5 \geq 0\) Always

\[\text{Soln: } (-\infty, \infty)\]

34) \(4x^2 + 12x < -9\)

\[4x^2 + 12x + 9 < 0\]

\[\text{solve: } 4x^2 + 12x + 9 = 0\]
\[4x^2 + 6x + 6x + 9 = 0\]
\[2x(2x+3) + 3(2x+3) = 0\]
\[(2x+3)(2x+3) = 0\]
\[2x + 3 = 0\]
\[x = -\frac{3}{2}\]

\[\begin{array}{ccccc}
& & & - & \quad + \\
-3 & & & \quad \frac{3}{2}
\end{array}\]

\[\text{Soln: } \emptyset\]
36.) \( x^3 - 9x^2 > 0 \)

Solve: \( x^3 - 9x^2 = 0 \)
\( x^2(x-9) = 0 \)
\( x = 0 \) or \( x = 9 \)

So \( x^3 < 9 \)

So \( x^3 < -8 \)

Solve: \( x^3 + 8 = 0 \)

\((x+2)(x^2-2x+4) = 0\)

Sum of Cubes

\( x + 2 = 0 \) or \( x^2 - 2x + 4 = 0 \)

\( x = -2 \)

\( b^2 - 4ac = -12 \)

No Real Soln.

So \( x < -2 \)

\( x > 9 \)

\( x < 0 \)
Solving Rational Inequalities:
When solving rational inequalities, follow these steps:
1. Get zero on one side of the inequality.
2. Ensure that there is only one fraction. That is, combine all rational expressions into one.
3. Find "markers" by solving the corresponding equation for the numerator and for the denominator.
4. If $\leq$ or $\geq$, then the markers for the numerator will have brackets: $\]$ or $[\]$.
   
   If $<$ or $>$, then the markers for the numerator will have parentheses: $\)$ or $($.
   
   Markers for the denominator must always have parentheses: $\)$ or $($.
5. Plot markers on the real number line. Check numbers before, between & after each marker. Indicate the sign of the region on the number line.
6. Use appropriate regions to find intervals that make the statement true. Give solutions in interval notation.

Ex. 1.7 p. 165
44.) $\frac{x-3}{x+1} > 0$

Num: $x-3 = 0$
$x = 3$

Denom: $x+1 = 0$
$x = -1$

Soln: $(-\infty, -1) \cup (3, \infty)$
46) \( \frac{3-2x}{4x+5} \geq 0 \)

Num: \( 3-2x = 0 \)
\( x = \frac{3}{2} \) \[ \text{[Red]} \]

Denom: \( 4x+5 = 0 \)
\( x = -\frac{5}{4} \) \[ \text{[Blue]} \]

\[ \begin{array}{ccccccc}
\text{---} & + & + & - & - & - & -
\end{array} \]

Sln: \( \left( -\frac{5}{4}, \frac{3}{2} \right] \)

Note: **NEVER** cross multiply over an inequality. Always subtract or add fractions to move them to the same side.

58) \( \frac{2x-3}{x+3} \leq 1 \)

\( \text{Num}: x-6=0 \)
\( x=6 \) \[ \text{[Green]} \]

\( \text{Denom}: x+3=0 \)
\( x=-3 \) \[ \text{[Black]} \]

\[ \begin{array}{ccccccc}
\text{---} & + & - & - & + & + & +
\end{array} \]

\( \begin{array}{c}
neg \frac{3}{6}
\end{array} \)

Sln: \( [-3, 6] \)

\( \frac{2x-3}{x+3} \leq 0 \)

\( \frac{2x-3-(x+3)}{x+3} \leq 0 \)

\( \frac{2x-3-x-3}{x+3} \leq 0 \)

\( \frac{x-6}{x+3} \leq 0 \)
62) \( \frac{x-1}{x+1} > \frac{x}{x-1} \)

\[
\frac{(x-1)(x-1) - x(x+1)}{(x+1)(x-1)} > 0
\]

\[
\frac{x^2 - 2x + 1 - x^2 - x}{(x+1)(x-1)} > 0
\]

\[
\frac{-3x + 1}{(x+1)(x-1)} > 0
\]

Num: \(-3x + 1 = 0\)

\[x = \frac{1}{3}\] (C)

Denom: \((x+1)(x-1) = 0\)

\[x = -1 \text{ or } x = 1\] (C)

S01h. \((-\infty, -1) \cup \left(\frac{1}{3}, 1\right)\)
60. \( \frac{x-2}{2x+1} < -1 \)

\[
\frac{x-2}{2x+1} + 1 < 0
\]

\[
\frac{(x-2) + (2x+1)}{2x+1} < 0
\]

\[
\frac{x-2 + 2x+1}{2x+1} < 0
\]

\[
\frac{3x-1}{2x+1} < 0
\]

**Num:** \( 3x-1 = 0 \) \( \Rightarrow x = \frac{1}{3} \)

**Denom:** \( 2x+1 = 0 \) \( \Rightarrow x = -\frac{1}{2} \)

\[
\text{S\text{o}l\text{n}: } (-1/2, 1/3)
\]

---

54. \( x - \frac{12}{x} > 1 \)

\[
\frac{x - \frac{12}{x}}{1} > 1
\]

\[
\frac{x^2 - 12}{x} > 0
\]

**Num:** \( x^2 - x - 12 = 0 \)

\( (x-4)(x+3) = 0 \)

\( x = 4 \text{ or } x = -3 \)

**Denom:** \( x = 0 \)

\[
\text{S\text{o}l\text{n}: } (-3,0) \cup (4,\infty)
\]

---

42. \( x^4 > 9 \)

\( x^4 - 9 > 0 \)

\( x^4 - 9 = 0 \)

\( (x^2)^2 - 3^2 = 0 \)

\( (x^2 - 3)(x^2 + 3) = 0 \)

\( x^2 - 3 = 0 \text{ or } x^2 + 3 = 0 \)

\( x^2 = 3 \text{ or } x^2 = -3 \)

\( x = \pm \sqrt{3} \text{ or } x = \pm i\sqrt{3} \)

Only real solutions:

\( x = \pm \sqrt{3} \)

\[
\text{S\text{o}l\text{n}: } (-\sqrt{3}, \sqrt{3}) \cup (\sqrt{3}, \infty)
\]