MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the absolute value equation.

1) \(3|x - 3| = 18\) \(\Rightarrow\) \(|x - 3| = 6\)
   \(\therefore\) \(x = -3\) \(\land\) \(x = 9\)
   A) \(\{3, -9\}\)  
   B) \(\{9, -3\}\)  
   C) \(\{3\}\)  
   D) \(\emptyset\)

2) \(|3x - 2| + 3 = 0\)
   \(|3x - 2| = -3\)
   A) \(\left\{-\frac{1}{3}\right\}\)  
   B) \(\left\{-\frac{1}{3}, -\frac{5}{3}\right\}\)  
   C) \(\left\{\frac{5}{3}, \frac{1}{3}\right\}\)  
   D) \(\emptyset\)

Solve the inequality.

3) \(|7x| \leq 2\)
   \(-2 \leq 7x \leq 2\) \(\Rightarrow\) \(-\frac{2}{7} \leq x \leq \frac{2}{7}\)
   A) \(\left[-\frac{2}{7}, \frac{2}{7}\right]\)  
   B) \(\left[-\frac{2}{7}, \frac{2}{7}\right]\)  
   C) \(\left[0, \frac{2}{7}\right]\)  
   D) \(\left[-\frac{7}{2}, \frac{7}{2}\right]\)

Solve the equation.

4) \(|n - 8| = |5 - n|\)
   A) \(\left\{\frac{13}{2}\right\}\)  
   B) \(\{13\}\)  
   C) \(\left\{-\frac{3}{2}\right\}\)  
   D) \(\emptyset\)

\(n - 8 = 5 - n\) \(\lor\) \(n - 8 = -(5 - n)\)
\[2n = 13\]
\[n = \frac{13}{2} \lor -8 = -5\]
GRAPHS OF EQUATIONS
Symmetry in a graph is when there are matching points on both sides of a dividing line (or point). The graph forms a “mirror image” about the line.
To get a point symmetric with respect to the $x$-axis, think of the mirror image or "flipping" it over the $x$-axis.

Let's look at the point $(6, 4)$.

So to be symmetric with respect to the $x$-axis, for the point $(x, y)$, you would get the point $(x, -y)$.

This is the point $(6, -4)$. 
To get a graph symmetric with respect to the $x$-axis, think of the mirror image or “flipping” it over the $x$-axis.

Let's look at a graph.

So to be symmetric with respect to the $x$-axis, for every point $(x, y)$ on the graph, the point $(x, -y)$ is also on the graph.
This is the point \((-6, 4)\).

Let's look at the point \((6, 4)\).

So to be symmetric with respect to the y-axis, for the point \((x, y)\), you would get the point \((-x, y)\).

To get a point symmetric with respect to the y-axis, think of the mirror image or "flipping" it over the y-axis.
Let's look at a graph

To get a graph symmetric with respect to the y-axis, think of the mirror image or "flipping" it over the y-axis.

So to be symmetric with respect to the y-axis, for every point \((x, y)\) on the graph, the point \((-x, y)\) is also on the graph.
So to be symmetric with respect to the origin, for the point \((x, y)\), you would get the point \((-x, -y)\).

Let's look at the point \((6,4)\).

To get a point symmetric with respect to the origin, think putting a push pin in the origin and rotating the graph \(180^\circ\) (like turning the paper over or standing on your head and looking at it).

This is the point \((-6, -4)\).
Let's look at a graph

To get a point symmetric with respect to the origin, think putting a push pin in the origin and rotating the graph 180° (like turning the paper over).

So to be symmetric with respect to the origin, for every point \((x, y)\) on the graph, the point \((-x, -y)\) is also on the graph.
Tests for Symmetry

Given an equation, you can test whether the graph of the equation has symmetry by the following:

<table>
<thead>
<tr>
<th>Axis</th>
<th>Test</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-axis</td>
<td>Replace $y$ by $-y$. After simplifying, if you get the original equation back it has x-axis symmetry</td>
<td></td>
</tr>
<tr>
<td>y-axis</td>
<td>Replace $x$ by $-x$. After simplifying, if you get the original equation back it has y-axis symmetry</td>
<td></td>
</tr>
<tr>
<td>origin</td>
<td>Replace $x$ by $-x$ and $y$ by $-y$. After simplifying, if you get the original equation back it has origin symmetry</td>
<td></td>
</tr>
</tbody>
</table>
Test for symmetry:

\[ y^2 - x - 4 = 0 \]

\[ y^2 - (-x) - 4 = 0 \]

\[ y^2 + x - 4 = 0 \]

So graph DOES NOT have y-axis symmetry

Replace \( x \) by \(-x\). After simplifying, if you get the original equation back it has y-axis symmetry.
Test for symmetry:

$$y^2 - x - 4 = 0$$

$$(-y)^2 - (-x) - 4 = 0$$

$$y^2 + x - 4 = 0$$

So graph **DOES NOT have** origin symmetry.

Replace $x$ by $-x$ and $y$ by $-y$. After simplifying, if you get the original equation back it has origin symmetry.
Finding $x$ and $y$ intercepts for a graph

The $x$-intercept is where a graph crosses the $x$-axis.

What is the common thing you notice about the $x$-intercepts of these graphs?

The $y$ value of the point where they cross the axis will always be 0.

To find the $x$-intercept when we have an equation then, we will want the $y$ value to be zero.
Now let's see how to find the $y$-intercept.

The $y$-intercept is where a line crosses the $y$-axis.

What is the common thing you notice about the $y$-intercepts of these lines?

The $x$ value of the point where they cross the axis will always be 0.

To find the $y$-intercept when we have an equation then, we will want the $x$ value to be zero.
Let's look at the equation $2x - 3y = 12$

Find the $x$-intercept. \[ 2x - 3(0) = 12 \]

We'll do this by plugging 0 in for $y$

\[ \frac{2x}{2} = \frac{12}{2} \]

Now solve for $x$.

$x = 6$

So the place where this line crosses the $x$-axis is (6, 0)
2x \ - \ 3y = 12

Find the y-intercept. We'll do this by plugging 0 in for x

2(0) \ - \ 3y = 12

\[ \frac{-3y}{-3} = \frac{12}{-3} \quad y = -4 \]

Now solve for y.

So the place where this line crosses the y-axis is (0, -4)

Since this was a linear equation, we could now graph the line
In this section we are going to use the point plotting method to graph equations. Look at the following equation.

\[ y = x^2 - 4 \]

Let's find \(x\) and \(y\) intercepts. The \(x\) intercept is where the \(y\) value is zero.

\[
\begin{align*}
0 &= x^2 - 4 \\
+4 &= +4 \\
4 &= x^2
\end{align*}
\]

Solving for \(x\)

Since \(x^2\) is 4, \(x\) could be 2 or \(-2\) so the \(x\) intercepts are (2,0) and (-2,0)

The \(y\) intercept is where the \(x\) value is zero.

\[
y = 0^2 - 4
\]

This means then that \(y = -4\) and the \(y\) intercept is (0,-4)
Ex. 2.2 (p. 202)
Find the intercepts of the given equation.

32.) \( \frac{x}{5} + \frac{y}{3} = 1 \)
   \[ \text{x-int: } \frac{x}{5} + \frac{0}{3} = 1 \]
   \[ \frac{x}{5} = 1 \]
   \[ x = 5 \quad (5, 0) \]

   \[ \text{y-int: } \frac{0}{5} + \frac{y}{3} = 1 \]
   \[ \frac{y}{3} = 1 \]
   \[ y = 3 \quad (0, 3) \]

36.) \( x = y^2 - 5y + 6 \)
   \[ \text{x-int: } x = 0^2 - 5(0) + 6 \]
   \[ x = 6 \quad (6, 0) \]

   \[ \text{y-int: } 0 = y^2 - 5y + 6 \]
   \[ 0 = (y-2)(y-3) \]
   \[ y = 2, \quad y = 3 \]
   \[ (0, 2), \quad (0, 3) \]

40.) \( xy = 1 \)
   \[ \text{x-int: } x(0) = 1 \]
   \[ 0 = 1 \quad \emptyset \]

   \[ \text{y-int: } (0)y = 1 \]
   \[ 0 = 1 \quad \emptyset \]

   \( \text{No intercepts.} \)

Determine the symmetries of the given function.

42.) \( x = y^2 + 1 \)
   \[ \text{x-axis: } x = (-y)^2 + 1 \]
   \[ x = y^2 + 1 \]
   \( \text{Yes} \)

   \[ \text{y-axis: } -x = y^2 + 1 \]
   \[ (-1) \]
   \[ x = -y^2 - 1 \]
   \( \text{No} \)

   \[ \text{origin: } -x = (-y)^2 + 1 \]
   \[ x = (y)^2 + 1 \]
   \( \text{No} \)

\( \text{Only x-axis symmetry} \)
44.) \( y = 2x^3 - x \)

\[ \text{y-axis: } y = 2(-x)^3 - (-x) \]
\[ \text{Origin: } y = 2(-x)^3 - (-x) \]
\[ \text{No } y = -2x^3 + x \]

\[ \text{No } y = -2x^3 + x \]
\[ \text{Yes } y = 2x^3 - x \]

\[ \text{Only Origin Symmetry} \]

46.) \( y = -3x^6 + 2x^4 + x^2 \)

\[ \text{y-axis: } y = -3(-x)^6 + 2(-x)^4 + (-x)^2 \]
\[ \text{Yes } y = -3x^6 + 2x^4 + x^2 \]

\[ \text{Origin: } y = -3(-x)^6 + 2(-x)^4 + (-x)^2 \]
\[ \text{No } y = -3x^6 + 2x^4 + x^2 \]

\[ \text{Only Y-axis Symmetry} \]

49.) \( x^2y^2 + 2xy = 1 \)

\[ \text{y-axis: } (-x)^2y^2 + 2(-x)y = 1 \]
\[ \text{No } x^2y^2 - 2xy = 1 \]

\[ \text{No } x^2y^2 - 2xy = 1 \]
\[ \text{Yes } x^2y^2 + 2xy = 1 \]

\[ \text{Only Origin Symmetry} \]
The standard form of the equation of a circle with its center at the origin is

\[ x^2 + y^2 = r^2 \]

\( r \) is the radius of the circle so if we take the square root of the right hand side, we'll know how big the radius is.

Notice that both the \( x \) and \( y \) terms are squared. Linear equations don’t have either the \( x \) or \( y \) terms squared. Parabolas have only the \( x \) term squared (or only the \( y \) term, but NOT both).
Let's look at the equation \( x^2 + y^2 = 9 \). This is \( r^2 \) so \( r = 3 \).

The center of the circle is at the origin and the radius is 3.

Let's graph this circle.

The circle is the set of all points that are 3 away (the radius) from the center. Let's count out 3 along each axis.

Center at (0, 0)
If the center of the circle is **NOT** at the origin then the equation for the standard form of a circle looks like this:

\[(x - h)^2 + (y - k)^2 = r^2\]

The center of the circle is at \((h, k)\).

\[(x - 3)^2 + (y - 1)^2 = 16\]

Find the center and radius and graph this circle.

The center of the circle is at \((h, k)\) which is \((3,1)\).

The radius is 4
If you take the equation of a circle in standard form for example:

\[(x + 2)^2 + (y - 4)^2 = 4\]

This is \(r^2\) so \(r = 2\)

Remember the center is at \((h, k)\) with \((x - h)\) and \((y - k)\) since the \(x\) is plus something and not minus, \((x + 2)\) can be written as \((x - (-2))\)

You can find the center and radius easily.
The center is at \((-2, 4)\) and the radius is 2.

But what if it was not in standard form but multiplied out (FOILED)

\[x^2 + 4x + 4 + y^2 - 8y + 16 = 4\]

Moving everything to one side in descending order and combining like terms we'd have:

\[x^2 + y^2 + 4x - 8y + 16 = 0\]
\[ x^2 + y^2 + 4x - 8y + 16 = 0 \]

If we'd have started with it like this, we'd have to complete the square on both the x's and y's to get in standard form.

Group x terms with a place to complete the square

\[ x^2 + 4x + \underline{\phantom{16}} + y^2 - 8y + \underline{\phantom{16}} = -16 + \underline{\phantom{4}} + \underline{\phantom{16}} \]

Move constant to the other side

Good idea to make blanks here so you don't forget to add to both sides when completing the square

Group y terms with a place to complete the square

Complete the square

Write factored and wahlah! back in standard form.

\[ (x + 2)^2 + (y - 4)^2 = 4 \]
Now let's work some examples:

Find an equation of the circle with center at (0, 0) and radius 7.

Let's sub in center and radius values in the standard form

\[(x-a)^2 + (y-b)^2 = r^2\]

\[(x-0)^2 + (y-0)^2 = 7^2\]

\[x^2 + y^2 = 49\]
Find an equation of the circle with center at \((0, 0)\) that passes through the point \((-1, -4)\).

Since the center is at \((0, 0)\) we'll have

\[ x^2 + y^2 = r^2 \]

The point \((-1, -4)\) is on the circle so should work when we plug it in the equation for \(x\) and \(y\):

\[
\begin{align*}
(-1)^2 + (-4)^2 &= r^2 \\
&= 1 + 16 = 17
\end{align*}
\]

Subbing this in for \(r^2\) we have:

\[ x^2 + y^2 = 17 \]

\[ r = \sqrt{17} \]
Find an equation of the circle with center at (-2, 5) and radius 6

Subbing in the values in standard form we have:

\[(x-h)^2 + (y-k)^2 = r^2\]

\[\left(x - -2\right)^2 + \left(y - 5\right)^2 = 6^2\]

\[(x+2)^2 + (y - 5)^2 = 36\]
Find an equation of the circle with center at (8, 2) and passes through the point (8, 0).

Subbing in the center values in standard form we have:

\[(x - h)^2 + (y - k)^2 = r^2\]

\[(x - 8)^2 + (y - 2)^2 = r^2\]

Since it passes through the point (8, 0) we can plug this point in for \(x\) and \(y\) to find \(r^2\).

\[(8 - 8)^2 + (0 - 2)^2 = r^2 = 4\]

\[(x - 8)^2 + (y - 2)^2 = 4\]
Identify the center and radius and sketch the graph:

\[
\frac{9x^2}{9} + \frac{9y^2}{9} = \frac{64}{9}
\]

To get in standard form we don't want coefficients on the squared terms so let's divide everything by 9.

\[
x^2 + y^2 = \frac{64}{9}
\]

So the center is at \((0, 0)\) and the radius is \(8/3\).
Identify the center and radius and sketch the graph:

\[(x + 4)^2 + (y - 3)^2 = 25\]

Remember the center values end up being the opposite sign of what is with the x and y and the right hand side is the radius squared.

So the center is at (-4, 3) and the radius is 5.
Find the center and radius of the circle:

\[ x^2 + y^2 + 6x - 4y - 3 = 0 \]

We have to complete the square on both the x’s and y’s to get in standard form.

Group \( x \) terms and a place to complete the square

\[ x^2 + 6x + \underline{9} + y^2 - 4y + \underline{4} = +3 + \underline{9} + 4 \]

Write factored for standard form.

\[ (x + 3)^2 + (y - 2)^2 = 16 \]

So the center is at (-3, 2) and the radius is 4.
Find the equation (in slope-intercept form) of the line that passes through the centers of the circles with the given equations:

\[ x^2 + y^2 - 6x + 10y - 4 = 0 \quad \text{and} \quad x^2 + y^2 + 8x - 16y + 2 = 0 \]

We need the slope and point.

We need the centers.

\[ x^2 - 6x + 9 + y^2 + 10y + 25 = 4 + 9 + 25 \]
\[ (x - 3)^2 + (y + 5)^2 = 38 \]

Center: \((3, -5)\)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-5)}{-4 - 3} = -\frac{13}{7} \]

Using \((3, -5)\):

\[ y - y_1 = m(x - x_1) \]
\[ y - (-5) = -\frac{13}{7} (x - 3) \]

\[ y = -\frac{13}{7} x + \frac{39}{7} - \frac{35}{7} \]

Center: \((-4, 8)\)

\[ x^2 + 8x + 16 + y^2 - 16y + 64 = -2 + 16 + 64 \]
\[ (x + 4)^2 + (y - 8)^2 = 78 \]

\[ y + 5 = -\frac{13}{7} x + \frac{39}{7} - \frac{5}{7} \]
Determine if the equation is a circle. If so, find the center and radius of the circle.

70.) \( x^2 + y^2 + 1 = 0 \)

\[
\begin{align*}
  x^2 + y^2 &= -1 \\
  \text{Not a Circle} \quad \left( x^2 \text{ is not positive.} \right)
\end{align*}
\]

68.) \( 3x^2 + 3y^2 + 6x = 0 \)

\[
\begin{align*}
  x^2 + y^2 + 2x &= 0 \\
  x^2 + 2x + 1 + y^2 &= 0 + 1 \\
  \text{Already a square} \\
  (x+1)^2 + (y-0)^2 &= 1 \\
  \text{Center: } (-1,0) \quad \text{radius} = 1
\end{align*}
\]

66.) \( x^2 + y^2 - 4x - 2y - 15 = 0 \)

\[
\begin{align*}
  x^2 - 4x + \frac{4}{4} + y^2 - 2y + \frac{1}{4} &= 15 + \frac{4}{4} + \frac{1}{4} \\
  (x-2)^2 + (y-1)^2 &= 20 \\
  \text{Center: } (2,1) \quad \text{radius} = \sqrt{20}
\end{align*}
\]

Find the center-radius form of the equation of the circle satisfying the given conditions.

62.) Center (1, 2); touching the x-axis

\[
\begin{align*}
  (x-1)^2 + (y-2)^2 &= r^2 \\
  (x-1)^2 + (y-0)^2 &= r^2 \\
  (1-1)^2 + (0-2)^2 &= r^2 \\
  4 &= r^2 \\
  \text{so:} \quad (x-1)^2 + (y-2)^2 &= 4
\end{align*}
\]
Find the center-radius form of the equation of the circle satisfying the given conditions.

64.) Diameter has endpoints (7, 4) and (-3, 6)

Center = Midpoint of Diameter = \( \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right) \)

\( (h, k) = \left( \frac{7+(-3)}{2}, \frac{4+6}{2} \right) = (2, 5) \)

\( (x-h)^2 + (y-k)^2 = r^2 \)

\( (x-2)^2 + (y-5)^2 = r^2 \)

Using (7, 4): \( (7-2)^2 + (4-5)^2 = r^2 \)  OR  Using (-3, 6):

\( (-3-2)^2 + (6-5)^2 = r^2 \)

\( 25 + 1 = r^2 \)  \( 25 + 1 = r^2 \)

\( 26 = r^2 \)

\( 26 = r^2 \)

Equation: \( (x-2)^2 + (y-5)^2 = 26 \)