Algebra Review for Trigonometry

Part I: Simplify the following completely.

1. \( \frac{1}{\sqrt{2}} \)  
2. \( \frac{2}{\sqrt{3}} \)  
3. \( \frac{\sqrt{2}}{2} \)  
4. \( \sqrt{(-1)^2 + (-3)^2} \)  
5. \( (\sqrt{2})^2 - (\sqrt{x-1})^2 + (\sqrt{5})^3 \)

Part II: Combine and simplify completely.

6. \( \pi + \frac{\pi}{3} \)  
7. \( 2\pi - \frac{\pi}{4} \)  
8. \( \pi - \frac{7\pi}{6} \)  
9. \( 2\pi + \frac{5\pi}{4} \)

Part III: Solve the following for \( x \).

10. \( 2x = \frac{3\pi}{4} + \pi n \)  
11. \( 3x = \pi + 2\pi n \)  
12. \( 4x - \frac{\pi}{4} = \frac{\pi}{2} + \pi n \)  
13. \( \frac{1}{3}x + \frac{\pi}{2} = \frac{\pi}{3} + 2\pi n \)

Part IV: Simplify the following completely. Don’t forget about Aunt Sally!

14. \( \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \)  
15. \( \frac{4}{3} + \left( \frac{5}{12} \right) \)  
16. \( 1 - \frac{\sqrt{3}}{3} \)  
17. \( \sqrt{1 - \frac{\sqrt{2}}{2}} \)  
18. \( \frac{-\sqrt{2}}{1 - \sqrt{2}} \)

Part V: Simplify the following completely. Don’t forget about Aunt Sally!

19. \( \frac{\frac{1}{2}}{1 - \frac{1}{4}} \)  
20. \( 2 \left( -\frac{1}{\sqrt{5}} \right) \left( -\frac{2}{\sqrt{5}} \right) \)  
21. \( \left( -\frac{2}{\sqrt{5}} \right)^2 - \left( -\frac{1}{\sqrt{5}} \right)^2 \)
Part VI: Simplify the following completely.

22. \( \frac{\pi}{2} + 2(1)\pi = \frac{\pi}{4} \)

23. \( \frac{\pi}{4} + 2(3)\pi = \frac{315 + 60(3)}{3} \)

Answers:

1. \( \frac{\sqrt{2}}{2} \)

2. \( \frac{2\sqrt{3}}{3} \)

3. \( \frac{\sqrt{6}}{4} \)

4. \( \sqrt{10} \)

5. \( 3 + 5\sqrt{5} - x \)

6. \( \frac{4\pi}{3} \)

7. \( \frac{7\pi}{4} \)

8. \(-\frac{\pi}{6} \)

9. \( \frac{13\pi}{4} \)

10. \( x = \frac{3\pi}{8} + \frac{\pi}{2} n \)

11. \( x = \frac{\pi}{3} + \frac{2\pi}{3} n \)

12. \( x = \frac{3\pi}{16} + \frac{\pi}{4} n \)

13. \( x = -\frac{\pi}{2} + 6\pi n \)

14. \( \frac{\sqrt{2} + \sqrt{6}}{4} \)

15. \( \frac{33}{56} \)

16. \( 2 - \sqrt{3} \)

17. \( -\frac{\sqrt{2} - \sqrt{2}}{2} \)

18. \( -\sqrt{2} - 1 \)

19. \( \frac{4}{3} \)

20. \( \frac{4}{5} \)

21. \( \frac{3}{5} \)

22. \( \frac{5\pi}{8} \)

23. \( \frac{25\pi}{12} \)

24. \( 465 \)
Common Angles in Trigonometry

The “45’s”

The “30’s and 60’s”

All the Common Angles
# Greek Alphabet Reference Guide

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"News from the Right Angle"

The mystery of the derivation of SOH CAH TOA, the acronym for trigonometric ratios, has finally been solved. Recently, investigators traced its true roots to a tribe called the Hypotens. This ancient clan had an amplitude of customs that modern civilization would consider odd. Only lately has it been discovered how these practices gave birth to SOH CAH TOA.

The Hypotens lived in dwellings that looked like lean-tos and that rested against trees, as illustrated. These homes were not shaped like hexagons or pentagons but simple three-sided polygons called "trigons." Often Hypotens would describe one of these homes as "trigon on my tree." Hence the land of the Hypotens became known as Trigonometry.

Another curious fact about the Hypotens was that they did not speak but instead used only "sine" language. They had very sensitive ears, and even the most subtle noise would cause them excruciating pain. Not only did they avoid making a sound, but they kept their hands over their ears. Since their hands were always occupied in this way, they had to sit and gesture with their feet when they wanted to talk. Naturally, communication was a difficult process.

Because talking was so arduous, the Hypotens published a daily periodical of events, which became known as the Hypoten News. The Hypoten News was not a very reliable journal because the Hypotens could not always agree on what was to be printed. The leaders of the two major political parties would meet daily to discuss the editorials and interpolations of the day's events. They would remain seated next to each other in adjacent seats. When this occurred, the cooperation signal was given, the "cosine," so named because the leaders were sitting "adjacent over the Hypoten News:"

\[
\text{Cosine} = \frac{\text{Adjacent}}{\text{Hypotenuse}}
\]

Now, this sequence continued for many years until a visitor from Argentina observed such a meeting and offered a suggestion. He pointed out that if they could not agree, the rational thing to do was simply to publish two bulletins, one for each of the divergent views. In other words, the "tangent" said that they could remain "opposite over adjacent" and still publish the Hypoten News:

\[
\text{Tangent} = \frac{\text{Opposite}}{\text{Adjacent}}
\]

Upon hearing this radical suggestion, the Hypotens fell into a violent argument. A change of such a hollowed tradition was bound to waggle a few toes! The Hypotens argued long into the night until their feet were exhausted. Not only did they never agree on the "tangent's" suggestion, but days of soaking their toes in the tub were required before the acute pain subsided and many of them could converse again.

From that day forward the famous argument has become known as the great "SOH CAH TOA."
Section 5.2 – Right Triangle Trigonometry - Notes

1) Right Triangle Trigonometry:

2) Special Triangles:

3) Cheating Chart:

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4) Fundamental Identities:

   A. Reciprocal Identities:

   B. Quotient Identities:

   Examples:

   C. Pythagorean Identities:

   D. Cofunction Identities:

5) Calculator Values:
6) Other Examples:

Ex 1:

Ex 2:

Ex 3:

Ex 4:

Ex 5:  

Angle of Elevation:  
Angle of Depression:

Sam is standing on a bench that is 2 feet from the ground. His eyes are 6 feet from the top of the bench. He sees a cat in a tree above with an angle of elevation of 28 degrees. The distance from his eyes to the cat is 50 feet. He sees a bug on the ground at an angle of depression of 41 degrees.

a) How high is the cat off the ground?

b) What is the distance from the base of the bench to the bug?
Some Mnemonics to Remember Your Trig Ratios

For years, students of mathematics have recalled the trigonometric ratios by remembering the Great Chief SOHCAHTOA. Each letter of the Chief’s name represents the name for one of the trig ratios of the sides of a right triangle:

\[ \text{Sine of an angle} = \frac{\text{Opposite}}{\text{Hypotenuse}} \]
\[ \text{Cosine of an angle} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \]
\[ \text{Tangent of an angle} = \frac{\text{Opposite side}}{\text{Adjacent}} \]

Some students may prefer the following mnemonics for recalling trig ratios:

- Some Of Her Children Are Having Trouble Over Algebra.
- Some Out-Houses Can Actually Have Totally Odorless Aromas.
- She Offered Her Cat A Heaping Teaspoon Of Acid.
- Stamp Out Homework Carefully, As Having Teachers Omit Assignments.
- Some Old Horse Caught Another Horse Taking Oats Away.
- Some Old Hippie Chasing Another Hippie Tripping On Apples.
- School! Oh How Can Anyone Have Trouble Over Academics?
- Saddle Our Horses, Canter Away Happily, To Other Adventures.
- Silly Old Henry Caught Albert Hugging Two Old Aunts.
- Some Old Hulks Carry A Huge Tub Of Ale
- Some Old Hag Cracked All Her Teeth On Asparagus
- Some Old Hairy Camels Are Hairier Than Others Are
- Smiles Of Happiness Come After Having Tankards Of Ale!!!
Section 5.8 – Applications of Trig Functions

1) Solving a Right Triangle Problems:

**Example 1:** Solve $\triangle ABC$ if $\beta = 24^\circ$, $c = 13.2$, and $\gamma = 90^\circ$.

**Example 2:** Solve $\triangle ABC$ if $a = 16.9$, $b = 42.7$, and $\gamma = 90^\circ$.

2) Angle of Elevation and Angle of Depression Problems:

Angle of Elevation: 

Angle of Depression:

**Example 3:** From a fourth floor balcony, Sandra spots Fred and Mary on the ground. The distance from the ground vertically to Sandra’s eyes is 125 feet. She sees Fred at a $52^\circ$ angle of depression and she sees Mary at a $24^\circ$ angle of depression. How far apart are Mary and Fred?
3) Bearing Problems:

Bearing / direction:

Example 4: You run from home at a bearing of N57°E for 3 miles. Your friend runs from your house S33°E for 6 miles. How far apart are you from your friend?

Example 5: Two ships leave the same port. Ship A travels N47°W at a speed of 12 mph. Two hours later, Ship B travels S43°W at a speed of 16 mph. What is the bearing from Ship B to Ship A four hours after Ship B left the port?
Fundamental Trigonometric Identities

Reciprocal Identities:
\[
\begin{align*}
\sin x &= \frac{1}{\csc x} & \csc x &= \frac{1}{\sin x} \\
\cos x &= \frac{1}{\sec x} & \sec x &= \frac{1}{\cos x} \\
\tan x &= \frac{1}{\cot x} & \cot x &= \frac{1}{\tan x}
\end{align*}
\]

Quotient Identities:
\[
\begin{align*}
\tan x &= \frac{\sin x}{\cos x} & \cot x &= \frac{\cos x}{\sin x}
\end{align*}
\]

Pythagorean Identities:
\[
\begin{align*}
\sin^2 x + \cos^2 x &= 1 & \tan^2 x + 1 &= \sec^2 x & \cot^2 x + 1 &= \csc^2 x
\end{align*}
\]

Even-Odd Identities:
\[
\begin{align*}
\sin(-x) &= -\sin x & \cos(-x) &= +\cos x & \tan(-x) &= -\tan x \\
\csc(-x) &= -\csc x & \sec(-x) &= +\sec x & \cot(-x) &= -\cot x
\end{align*}
\]

Cofunction Identities:
\[
\begin{align*}
\sin(x) &= \cos(90^\circ - x) & \cos(x) &= \sin(90^\circ - x) \\
\tan(x) &= \cot(90^\circ - x) & \cot(x) &= \tan(90^\circ - x) \\
\csc(x) &= \sec(90^\circ - x) & \sec(x) &= \csc(90^\circ - x)
\end{align*}
\]
Here is Michael Stuben's effort to introduce the trigonometry sum and difference formulas in an interesting way:

As we all know, some of the people to whom we are attracted are not attracted to us. And it is not unusual for a person who has shown interest in us to later lose interest in us. Maybe that is a good thing, because it forces us to date a lot of people and to become more experienced in maintaining relationships.

Anyway, this is the story of Sinbad and Cosette. Sinbad loved Cosette, but Cosette did not feel the same way about Sinbad. Naturally, when Sinbad was in charge of their double date, he put himself with Cosette, and he put his brother with her sister:

\[
\sin(A + B) = \sin A \cos B + \cos A \sin B.
\]
\[
\sin(A - B) = \sin A \cos B - \cos A \sin B.
\]

Sinbad loved to tell people that his and Cosette's signs were the same.

However, when Cosette was in charge of the double date she placed herself with her sister and put Sinbad with his brother. She made sure everyone knew that their signs were NOT the same:

\[
\cos(A + B) = \cos A \cos B - \sin A \sin B.
\]
\[
\cos(A - B) = \cos A \cos B + \sin A \sin B.
\]

Also, notice that Cosette placed herself and her sister BEFORE Sinbad and his brother. This detail was important to Cosette. She was very snobby, you know.
Example 1: Solve $\triangle ABC$ if $\alpha = 43^\circ$, $\gamma = 54^\circ$ and $b = 58$.

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Example 2: Solve $\triangle ABC$ if $\alpha = 31^\circ$, $\beta = 102^\circ$ and $c = 10.6$.

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Example 3: Solve $\triangle ABC$ if $\alpha = 32.32^\circ$, $c = 574.3$ and $a = 263.6$.

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Example 4: Solve $\triangle ABC$ if $\alpha = 27'30''$, $c = 52.8$ and $a = 28.1$.

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Example #5: Approximate the area of \( \Delta ABC \) if \( a = 15.3 \text{ in} \), \( c = 20.7 \text{ in} \) and \( \beta = 53.8^\circ \).

Example #6: Approximate the area of \( \Delta ABC \) if \( b = 7 \text{ cm} \), \( c = 9 \text{ cm} \) and \( \gamma = 42^\circ \).

Example #7: (Like #58 in the text) A cable car carries passengers from Point A to Point C on Siegel Mountain. Point A is 1.6 miles from the base. The angle of elevation from Point A to the peak is \( 32^\circ \) and the angle of elevation from Point B to the peak is \( 66^\circ \).

a) What is the distance covered by the cable car from Point A to the peak? Answer in the nearest tenth of a foot.

b) What is the distance from Point B to the peak? Answer in the nearest tenth of a foot.

c) Use the right triangle to find the height of the mountain from the base to the peak to the nearest tenth of a foot.
Section 7.2 – The Law of Cosines – Notes

The Law of Cosines:

Example 1: Solve \( \Delta ABC \) if \( a = 5.2, c = 9.4 \) and \( \beta = 65^\circ \).

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Example 2: Solve \( \Delta ABC \) if \( a = 4, b = 10 \) and \( c = 12 \).

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Example 3: (#50 in text) – A Little League baseball diamond with four bases forms a square with each side measuring 60 feet. The pitcher’s mound is 46 feet from home plate on a line drawn from home plate to second base. What is the distance from the pitcher’s mound to third base (to the nearest tenth of a foot)?

Heron’s Formula:

Example 4: Two hikers leave the same spot, one traveling $N41^\circ E$ for 3.2 miles and the other traveling $S74^\circ W$ for 5.4 miles.

a) What is the distance between the hikers now?

b) What is the area of the triangle formed between the two hikers and where they started traveling?